

FACULTY OF PURE AND APPLIED MATHEMATICS SUBJECT CARD Name in Polish: Układy Dynamiczne i Teoria Ergodyczna Name in English: Dynamical Systems and Ergodic Theory Main field of study: Specialization (if applicable): Level and form of studies: 3rd level Kind of subject: general course Subject code: MAT1306 Group of courses: YES / NO*
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	Lecture	Classes	Laboratory	Project	Seminar
Number of hours of organized classes in University (ZZU)	30				
Number of hours of total student workload (CNPS)	90				
Form of crediting	Examination / crediting with grade*				
For group of courses mark (X) final course					
Number of ECTS points	3				
including number of ECTS points for practical (P) classes	2				
including number of ECTS points for direct teacher-student contact (BK) classes	1				

PREREQUISITES RELATING TO KNOWLEDGE, SKILLS AND OTHER COMPETENCES

1. Basic knowledge and abilities in the areas of Topology, Measure Theory and Functional Analysis.
2. Student is able to study supplementary areas of knowledge and skills.

SUBJECT OBJECTIVES

- C1 Students will learn selected notions of the Theory of Dynamical Systems including Ergodic Theory, and main theorems concerning these notions.
- C2 The student will recognize the connections between the notions of the Theory of Dynamical Systems and notions from other areas of mathematics, such as Stochastic Processes, Differential Equations, or Number Theory.
- C3 The student should acquire the skills of applying selected tools of the Theory of Dynamical Systems to solving problems, including problems from other areas of mathematics and physics.

SUBJECT EDUCATIONAL EFFECTS

relating to knowledge:

PEK_W01 Acquires the knowledge concerning the methods of the Theory of Dynamical Systems

PEK_W02 Knows the connections and applications of the above notions with other areas of mathematics

relating to skills:

PEK_U01 Skills and ability connected to the scientific methodology of research

PEK_U02 Skills and ability in creating and self-sufficient conduct of scientific research

relating to social competences:

PEK_K01 awareness of the role of interdisciplinary collaboration

PEK_K02 awareness of the role of popularization of science

PROGRAMME CONTENT

Form of classes - lecture		Number of hours
Lec1	Abstract notion of a dynamical system with the action of a group or subgroup, examples	2
Lec2	Topological dynamical systems, transitivity, minimality, recurrence, factors, conjugacy, proximal pairs, asymptotic pairs, Li-Yorke chaos	2
Lec3	Symbolic systems. Properties, characterization, applications	2
Lec4	Connections with Number Theory – IP sets and other. Mention of Sarnak Conjecture	2
Lec5	Measure-preserving transformations. Factors and isomorphism. Recalling the basic theorems: Poincare recurrence, Rokhlin Lemma and Ergodic Birkhoff Theorem. Connections with stochastic processes.	2
Lec6	Invariant measures in a topological system. Simplex of measures. Mention of the Furstenberg Conjecture.	2
Lec7-8	Spectral properties of a dynamical system. Koopman Operator and Markov Operator. Spectral Theorem. Point Spectrum, pure point spectrum (measure theoretic and topological). Equicontinuity. Maximal equicontinuous factor, Kronecker factor. Group rotation. Halmos – von Neumann Theorem.	4
Lec9	Mixing, weak mixing, Bernoulli systems.	2
Lec10-12	Various constructions of dynamical systems: induced maps, suspensions, skew products, group extensions, semicycloid extensions, special flows, elements of the theory of Smooth Systems (Smale horseshoe).	6
Lec13	Topological and measurable joinings. Disjointness	2
Lec14	Topological and measure-theoretic entropy. Conditional entropy. Subadditivity.	2
Lec15	Applications of entropy, Variational principle, Ornstein Theorem.	2
	Total hours	30

TEACHING TOOLS USED

N1 lecture
N2 consultations
N3 written assignments: problem solutions

EVALUATION OF SUBJECT EDUCATIONAL EFFECTS ACHIEVEMENT

Evaluation (F – forming (during semester), P – concluding (at the semester end))	Educational effect number	Way of evaluating educational effect achievement
F1	PEK_U01, PEK_U02	participation in the course
F2	PEK_W01, PEK_W02, PEK_U01, PEK_U02, PEK_K01, PEK_K02	solutions of the problems
$P=0.5*F1+0.5*F2$		

PRIMARY AND SECONDARY LITERATURE

PRIMARY LITERATURE:

- [1] Peter Walters: An Introduction to Ergodic Theory, Springer 1982
- [2] Karl Petersen: Ergodic Theory
- [3] J. de Vries: Elements of Topological Dynamics

SECONDARY LITERATURE:

- [4] T. Downarowicz: Entropy in Dynamical Systems, Cambridge University Press 2011

SUBJECT SUPERVISOR (NAME AND SURNAME, E-MAIL ADDRESS)

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**MATRIX OF CORRELATION BETWEEN EDUCATIONAL EFFECTS FOR SUBJECT
DYNAMICAL SYSTEMS AND ERGODIC THEORY
AND EDUCATIONAL EFFECTS FOR MAIN FIELD OF STUDY**

Subject educational effect	Correlation between subject educational effect and educational effects defined for main field of study and specialization (if applicable)**	Subject objectives***	Programme content***	Teaching tool number***
PEK_W01	I3_W06	C1,C2	Lec1-15	N1,N2,N3
PEK_U01	I3_W06	C1,C2	Lec1-15	N1,N2,N3
PEK_U02	I3_U02	C2	Lec1-15	N2,N3
PEK_U03	I3_U05	C2,C3	Lec1-15	N2,N3
PEK_K01	I3_K01	C3	Lec1-15	N2,N3
PEK_K02	I3_K04	C3	Lec1-15	N2,N3

** - enter symbols for main-field-of-study/specialization educational effects

*** - from table above