

**FACULTY OF PURE AND APPLIED MATHEMATICS  
SUBJECT CARD**

Name in Polish: **Optymalne sterowanie**

Name in English: **Optimal control**

Main field of study (if applicable): **APPLIED MATHEMATICS**

Specialization (if applicable): **MODELLING, SIMULATION, OPTIMIZATION**

Level and form of studies: **1st/ 2nd\* level, full-time / part-time\***

Kind of subject: **obligatory / optional / ~~university-wide\*~~**

Subject code **MAT001586**

Group of courses **YES / NO\***

	Lecture	Classes	Laboratory	Project	Seminar
Number of hours of organized classes in University (ZZU)	30		30		
Number of hours of total student workload (CNPS)	150				
Form of crediting	Examination / <del>crediting with grade*</del>	Examination / crediting with grade*	Examination / crediting with grade*	Examination / crediting with grade*	Examination / crediting with grade*
For group of courses mark (X) final course	X				
Number of ECTS points	5				
including number of ECTS points for practical (P) classes	1		3		
including number of ECTS points for direct teacher-student contact (BK) classes	3				

**PREREQUISITES RELATING TO KNOWLEDGE, SKILLS AND OTHER COMPETENCES**

1. The student has basic knowledge of calculus, algebra and the probability theory.

**SUBJECT OBJECTIVES**

- C1 Understanding the concepts and methods of control.
- C2 Understanding the wording optimal control tasks.
- C3 Knowledge of the backgrounds for the analysis of dynamic systems.
- C4 Understanding models and analysis of stochastic control systems.
- C5 Application of acquired knowledge to create and analyze mathematical models to solve theoretical and practical problems in various fields of science and technology.

**SUBJECT EDUCATIONAL EFFECTS**

*The scope of the student's knowledge:*

**PEK\_W01:** Student knows the formulation of operational research problems.

**PEK\_W02:** He recognizes situations that require the application of operations research methods to solve practical problems.

**PEK\_W03:** He knows the limitations of analytical methods and the possibility of numerical analysis of dynamic models.

**PEK\_W04:** He knows the stochastic methods in operations research.

*The scope of the student's skills:*

- PEK\_U01. Student is able to formulate modeling task for analysis in a convenient form.
- PEK\_U02. He can use the appropriate algorithm to solve tasks in the operational research.
- PEK\_U03. Student is able to recognize issues that competent optimization methods are based on the use of stochastic camera.

*The scope of the student's social skills:*

- PEK\_K01 The student is able to find and use the recommended literature course and independently acquire knowledge
- PEK\_K02 The student is able to use the basic tools for the analysis of mathematical models
- PEK\_K03 The student understands the need for systematic and independent work on mastery of course material.

Course content		
Form of activities - lectures		Hours load
Wy1	Deterministic control system with discrete time. Algorithm of dynamic programming.	2
Wy2	Processes with discrete time. Markov chains. Conditional expectation. Martingales and Markov times.	2
Wy3	Markov decision processes. Bellman equation.	2
Wy4	Introduction to models with infinite horizon. Markov decision models with discounted payments, minimizing the average cost per unit and other criteria.	4
Wy5	Applications Markov decision processes in the reliability theory, the renewal theory, the queue theory.	2
Wy6	Optimal control of the continuous time. The Hamilton-Jacobi-Bellman equation.	2
Wy7	Linear systems with quadratic cost function and a complete state observation. The inventory control systems.	2
Wy8	Systems with uncertain state observation. Iterative determination of the value functions.	2
Wy9	The approximated solution of the Bellman equation.	2
Wy10	Optimal stopping of finite sequences.	2
Wy11	Optimal stopping of finite Markov sequences. Examples.	2
Wy12	Infinite horizon optimal stopping problem.	2
Wy13	The disorder detection problem.	2
Wy14	Suboptimal solutions of operation models. Adaptive systems.	2
<b>Total load (in hours)</b>		<b>30</b>

Form of activities: classes, practice		Number of hours
Ćw1	Examples of deterministic control systems with discrete time.	2
Ćw2	Properties of Markov chains and their analysis. Checking stationarity and ergodicity of stochastic sequences. Classification of states. Conditional expectation. Martingales and Markov moments.	2
Ćw3	Markov decision process for selected practical problems. Analysis of the Bellman equation for the constructed MDPs.	2

Ćw4	Investigation of infinite horizon models. Markov decision models with discounted payoffs, the average cost per unit, and other criteria.	4
Ćw5	Applications Markov decision processes in the reliability theory, the renewal theory, the queue theory-examples.	2
Ćw6	Optimal control of the continuous time. The Hamilton-Jacobi-Bellman equation.	2
Ćw7	Linear systems with quadratic cost function and a complete state observation. The inventory control systems.	2
Ćw8	Systems with uncertain state observation. Iterative determination of the value functions.	2
Ćw9	The approximated solution of the Bellman equation.	2
Ćw10	Optimal stopping of finite sequences.	2
Ćw11	Optimal stopping of finite Markov sequences. Examples.	4
Ćw12	Analysis of selected disorder problems.	2
Ćw13	Suboptimal solutions. Adaptive systems.	2
	Total hours	<b>30</b>

### TOOLS FOR TEACHING

- 1 Lecture - traditional method.  
2 Exercise and accounting problems - the traditional method.  
3 Consultation.  
4 Student's own work - preparing to exercise and test.

### EVALUATION OF SUBJECT EDUCATIONAL EFFECTS ACHIEVEMENT

Evaluation (F – forming (during semester), P – concluding (at semester end))	Educational effect number	Way of evaluating educational effect achievement
F1	PEK_W01, PEK_W02, PEK_W03 PEK_W04, PEK_W05 PEK_K01, PEK_K02	oral presentations, quizzes, tests
F2	PEK_W01, PEK_W02, PEK_W03 PEK_W04, PEK_W05 PEK_U01, PEK_U02, PEK_U03, PEK_U04 PEK_K01, PEK_K02, PEK_K03	exam
$P=0,4*F1+0,6*F2$		

### PRIMARY AND SECONDARY LITERATURE

#### **PRIMARY LITERATURE:**

- [1] Dimitri P. Bertsekas, Dynamic Programming and Optimal Control, vol. 1, Athena Scientific, Belmont, MA: 2005.
- [2] Dimitri P. Bertsekas, Dynamic Programming and Optimal Control, vol. 2, Athena Scientific, Belmont, MA: 2007.
- [3] Harold Kushner: Wprowadzenie do teorii sterowania stochastycznego. WNT, 1983.
- [4] A.N. Shiryaev. Optimal Stopping Rules. Springer-Verlag, New York, Heidelberg, Berlin, 1978.

**SECONDARY LITERATURE:**

- [1] J. P. Aubin, Optima and Equilibria. An Introduction to Nonlinear Analysis, Springer, Berlin 1993.
- [2] Wayne I. Winston: introduction to mathematical programming: applications and algorithms, 1991.

**SUBJECT SUPERVISOR (NAME AND SURNAME, E-MAIL ADDRESS)**

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MATRIX OF CORRELATION BETWEEN EDUCATIONAL EFFECTS FOR SUBJECT  
**Optimal Control MAT001586**  
 AND EDUCATIONAL EFFECTS FOR MAIN FIELD OF STUDY APPLIED  
 MATHEMATICS AND SPECIALIZATION MODELLING, SIMULATION,  
 OPTIMIZATION

AND SPECIALIZATION ..... Subject educational effect	Correlation between subject educational effect and educational effects defined for main field of study and specialization (if applicable)**	Subject objectives***	Programme content***	Teaching tool number***
PEK_W01 PEK_W02 PEK_W03 PEK_W04	K2MST_W01, K2MST_W02, K2MST_W03, K2MST_W06, K2MST_W07, K2MST_W08, K2MST_W10 K2MST_mso_W01 K2MST_mso_W02 K2MST_mso_W03	C1—C6	Wy1—Wy9	1,3,4
PEK_U01 PEK_U02 PEK_U03	K2MST_U01, K2MST_U02, K2MST_U03, K2MST_U15, K2MST_U16, K2MST_U17, K2MST_U18, K2MST_U19 K2MST_U24 K2MST_U25 K2MST_mso_U01 K2MST_mso_U02 K2MST_mso_U03	C1—C5	Ćw1-Ćw9	2,3,4
PEK_K01 PEK_K02 PEK_K03	K2MST_K0, K2MST_K02, K2MST_K03, K2MST_K04, K2MST_K05, K2MST_K06 K2MST_K07 K2MST_mso_K01 K2MST_mso_K02	C1, C2, C3, C4, C5, C6	Wy1-Wy14, Ćw1-Ćw9	1, 2, 3, 4

\*\* - from table above