COURSES TAUGHT IN ENGLISH (MASTER LEVEL)
FACULTY OF PURE AND APPLIED MATHEMATICS
WROCLAW UNIVERSITY OF SCIENCE AND TECHNOLOGY

L	t	lab	p	s

L – Lecture, **t** – Tutorials, **lab** – laboratory, **p** – project, **s** – seminar,

CHS	TSW

CHS – Contact Hours (organized), **TSW** – Total Student Workload (h),

E – Exam, T – Test, CW – Course Work

Winter Semester 2017/2018

Subject/Module	Contact hours/week					CHS	TSW	ECTS	Form of Assessment
	L	t	lab	p	s				
Optimization theory	2	2				60	180	6	E
Agent-based modelling of complex systems	2		2			60	150	5	E
Diploma Seminar					2	30	60	2	T/CW

Summer semester 2017/2018

Subject/Module	Contact hours/week					CHS	TSW	ECTS	Form of Assessment
	L	t	lab	p	s				
Economathematics	2	2				60	150	5	E
Partial differential equations with applications in physics and industry	2	2				60	180	6	E
Life Insurance Models	2	2				60	150	5	E

Optional courses (only 3 courses per semester are usually chosen)

Subject/Module	Contact hours/week					CHS	TSW	ECTS	Form of Assessment
	L	t	lab	p	s				
Financial risk management	2	2				60	150	5	T
Applied Functional analysis	2		2			60	150	5	T
Advanced Topics in Dynamic Games	2	2				60	150	5	T
Risk management in insurance	2			2		60	150	5	T
Insurance models for industry	2		2			60	150	5	T
Nonlinear Methods	2		2			60	150	5	T
Optimal control	2		2			60	150	5	T
Numerical methods in differential equations	2		2			60	150	5	T
Diffusion processes on complex networks	2		2			60	150	5	T
Statistical packages	2		2			60	150	5	T
Queues and Communication Networks	2	2				60	150	5	T
Computational Finance	2		2			60	150	5	T
Introduction to Big Data Analytics	2			2		60	150	5	T
Introduction to applied fluid dynamics	2			2		60	150	5	T
Mathematical Image Processing	2		2			60	150	5	T
Estimation Theory	2		2			60	150	5	T
Perturbation Methods	2		2			60	150	5	T
Analysis of unstructured data	2			2		60	150	5	T

Reserves in life and non-life insurance	2	2				60	150	5	T
Free boundary problems	2	2				60	150	5	T
Introduction to Inverse Problems	2		2			60	150	5	T
Operations Research	2		2			60	150	5	T
Computer simulations of stochastic processes	2		2			60	150	5	T

CONTENT OF THE COURSES TAUGHT IN ENGLISH (MASTER LEVEL)
FACULTY OF PURE AND APPLIED MATHEMATICS
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OPTIMIZATION THEORY

Lectures: Introduction to mathematical programming. Optimization without constraints. Global and local extremes. Optimality conditions. Gradient methods. Steepest descent method. Newton's method and its variants. Analysis of convergence. Linear programming. Geometric interpretation. Simplex algorithm. Dual problem. Duality theory for linear programming. Sensitivity analysis. Integer programming. Linear programming relaxation. Branch and bound method. The theory of Lagrange multipliers. The necessary and sufficient conditions for extreme for constraints in the equality form. Lagrange multipliers in sensitivity analysis. Constraints in the form of inequality. Optimality conditions of KarushKuhn-Tucker. Quadratic programming. Wolfe's algorithm. Elements of convex analysis. Projection theorem. Supporting hyperplane theorem. Separating hyperplane theorem. Optimization on a convex set. Frank-Wolfe's method. Gradient projection method. Convex programming. Duality for convex programming. Subgradient. Subgradient methods.
Classes: Necessary and sufficient optimality conditions. Properties of convex functions and convex sets. Illustration of gradient methods. Simplex method. Practical applications of linear programming. Sensitivity analysis. Applications of Lagrange multiplier theory in practical optimization problems. Quadratic programming problems. Applications of convex analysis in solving optimization problems.

AGENT-BASED MODELLING OF COMPLEX SYSTEM

Lectures: Introduction to agent-based modelling. Creating simple agent-based models. Exploring and extending agent-based models. Components of agent-based models. Analyzing agent-based simulations. Verification and validation of agent-based models. Computational roots of agent-based modelling. Models of natural and social complex systems – examples.
Lab: Practical introduction to Python modules for agent-based modelling. Practical introduction to Netlogo, Simple agent-based models (life, ant, heroes and cowards models). Analysis of existing models (fire, segregation and El Farol models). SI epidemics model – implementation and analysis. SIR epidemics model – implementation and analysis. Voter and q-voter models – implementation and analysis.

DIPLOMA SEMINAR

Seminar: Master thesis results presentations.

ECONOMATHEMATICS

Lectures: Black-Scholes model. Stochastic calculus and its application to the valuation of assets and liabilities and design hedging strategies. Feynman-Kac formula and Blacka-Scholes formula. Bachelier model. Risk-Neutral and Real World scenarios, deflator and its applications. Modeling of term structure. Vasicek and Cox-Ingerson-Ross models, HJM model, LIBOR model. Calibration of interest rate instruments. Valuation of debt instruments and interest rate derivatives (bonds, cap/ floor, caplet/floorlet and swaptions). Subdiffusive Black-Scholes and Bachelier models. Fractional Brownian motion in finance. Gerber-Shiu model, Esscher transform.
Classes: Illustration of all models. Analytical and computer methods. Examples of pricing derivatives.

PARTIAL DIFFERENTIAL EQUATIONS WITH APPLICATIONS IN PHYSICS AND INDUSTRY

Lectures: A reminder of information concerning first order partial differential equations. Methods of characteristics, weak solutions and shock waves. Second order partial differential equations and their classification. Physical motivations. Parabolic equations and their applications (heat, diffusion). Initial-boundary problems, method of separation of variables, Fourier transform, fundamental solution, maximum principle. Hyperbolic equations and their applications (vibration of strings, membranes and beams; acoustical, mechanical and electromagnetic waves). D'Alembert's solution, initial-boundary problems, method of separation of variables, Kirchhoff's solution, Huygens' principle. Elliptic equations and their applications (stationary temperature distribution, gravitational and electrostatic potential). Boundary value problems, eigenfunctions, Poisson's equation, Green's function. The calculus of variations and its applications. Euler-Lagrange equation, Lagrangian mechanics, geodesic equation, minimal surface equation.

Classes: Solving of problems for differential equations and their applications.

LIFE INSURANCE MODELS

Lectures: Distribution of the future lifetime including probability of survival and death, force of mortality. Life tables. Assumptions for fractional ages . Analytical laws of mortality. Multiple state models with estimation methods of their parameters and estimation methods of future lifetime (including Nelson-Aalen and Kaplan-Meier estimators). Life insurance payable at the moment death and at the end of the year of death. Discrete and continuous annuities. Net premiums in fully discrete and continuous insurance contracts. Commutation functions. Gross premiums. Pension funds.

Classes: Solving of problems illustrating theory given in the lectures, solving of problems from actuarial exams.

FINANCIAL RISK MANAGEMENT

Lectures: Fundamental theorems of asset pricing – overview. Greek parameters, delta/gamma hedging. Volatility modeling. Exotic options – overview. Stochastic control. Risk measures and financial risk. Portfolio pricing. Construction of optimal portfolio, effectiveness measures of investment portfolio. Measuring of default, asset and liability management and hedging strategies, immunization. Credit risk management. Operational risk management. Time variation in risk. Backtesting and stress testing.

Classes: Illustration of all models. Analytical and computer methods. Examples of pricing derivatives.

APPLIED FUNCTIONAL ANALYSIS

Lectures: Introduction to functional analysis – real world problems modeled by operator equations. Elements of topology and linear spaces. Linear normed spaces. Hilbert spaces. Linear operators. Elements of spectra theory. Fundaments of optimisation. Role of functional analysis in solving inverse problems. Elements of functional analysis in numerical methods.

Lab: Solving of problems illustrating theory given in the lectures using mathematical packages for numerical computing

ADVANCED TOPICS IN DYNAMIC GAMES

Lectures: Introduction to markovian decision processes, the concept of a policy, different optimality criteria, examples of simple models. Dynamic programming method. Solving models with finite time horizon. Backward induction. Models with infinite time horizon. The Banach fixed point theorem and its application to a solution of the Bellman equation. Algorithms applied to infinite time horizon models: value iteration, policy improvement, LP. Markov decision processes with risk sensitive payoff criteria. Other payoff criteria. Specific models. Two-person zero-sum discounted stochastic games. The theorem of Shapley. Nonzero-sum discounted stochastic games. Stochastic games with other payoff criteria. Applications of stochastic games in economics and engineering. Mean field games. The existence of solutions. Relation with games with a finite number of players. Examples of applications in economics and engineering.

Classes: Markov chains. Solving different markovian decision models. Solving different stochastic game models.

RISK MANAGEMENT IN INSURANCE

Lectures: Risk management in insurance, actuarial function, risk management function. Capital management, risk appetite, risk measures (including RAROC, RORAC). Solvency II: capital requirements, standard formula, internal models, risk categories. Profitability and risk exposure tests, monitoring of actuarial assumptions or parameters. Risk exposure reduction methods, methods and instruments of risk transfer including alternative risk transfers (ART). Proportional and non-proportional reinsurance as method of risk exposure reduction. Actuarial pricing in life and non-life insurance, risk factors. Application of derivatives in insurance. Pricing of catastrophe bonds.

Project: Preparation and presentations of projects illustrating theory given in the lectures.

INSURANCE MODELS FOR INDUSTRY

Lectures: Types of insurance policies in industry. Solvency II in Non-Life Insurance. Premium principles, risk measures. Franchises and their types. Pricing of net premiums with franchise. Individual risk model. Approximations for total loss in individual risk model. Collective risk model. Frequency and severity distributions of claims. Parameters and distributions of aggregate claim amount. Compound Poisson model. Practical consequences of the theorem on the sum of compund Poisson risk. The (a,b) class of distribution. Mixed Poisson model. Risk proces. The adjustment coefficient. The probability of ruin. Distribution of the maximal aggregate coefficient and ruin probability. Pollaczek-Khinchin formula. Approximations of ruin probability in finite and infinite time horizon. System Bonus-Malus. Credibility theory.

Lab: Solving of problems illustrating theory given in the lectures.

NONLINEAR METHODS

Lectures: Examples of nonlinear phenomena. Nonlinear oscillators. Bifurcation and stability. Van der Pol equation. Duffig equation. 2-D systems of nonlinear equations – equilibrium points. Classification of the equilibrium points. Systems of nonlinear equations – attractors. Lorenc equation. Strange attractors. Belolusov-Zabotynski equation. Benard cells – equations of hydrodynamics. Examples of nonlinear optimisation. Some methods of nonlinear optimisation.
Lab: Solving of problems illustrating theory given in the lectures by analytic methods and with MATLAB.

OPTIMAL CONTROL

Lectures: Deterministic control system with discrete time. Algorithm of dynamic programming. Processes with discrete time. Markov chains. Conditional expectation. Martingales and Markov times. Markov decision processes. Bellman equation. Introduction to models with infinite horizon. Markov decision models with discounted payments, minimizing the average cost per unit and other criteria. Applications Markov decision processes in the reliability theory, the renewal theory, the queue theory. Optimal control of the continuous time. The Hamilton-Jacobi-Bellman equation. Linear systems with quadratic cost function and a complete state observation. The inventory control systems. Systems with uncertain state observation. Iterative determination of the value functions. The approximated solution of the Bellman equation. Optimal stopping of finite sequences. Optimal stopping of finite Markov sequences. Examples. Infinite horizon optimal stopping problem. The disorder detection problem. Suboptimal solutions of operation models. Adaptive systems.

Lab: Examples of deterministic control systems with discrete time. Properties of Markov chains and their analysis. Checking stationarity and ergodicity of stochastic sequences. Classification of states. Conditional expectation. Martingales and Markov moments. Markov decision process for selected practical problems. Analysis of the Bellman equation for the constructed MDPs. Investigation of infinite horizon models. Markov decision models with discounted payoffs, the average cost per unit, and other criteria. Applications Markov decision processes in the reliability theory, the renewal theory, the queue theory-examples. Optimal control of the continuous time. The Hamilton-Jacobi-Bellman equation. Linear systems with quadratic cost function and a complete state observation. The inventory control systems. Systems with uncertain state observation. Iterative determination of the value functions. The approximated solution of the Bellman equation. Optimal stopping of finite sequences. Optimal stopping of finite Markov sequences. Examples. Analysis of selected disorder problems. Suboptimal solutions. Adaptive systems.

NUMERICAL METHODS IN DIFFERENTIAL EQUATIONS

Lectures: Recalling basic facts of theory of ordinary differential equations. Explicit and implicit Euler method of approximate solving of ordinary differential equations and their systems. Runge-Kutta type methods and other schemes of approximation of ordinary differential equations and their systems. Multistep methods, stability of numerical methods. Stiff problems. Methods of approximation of boundary value problems for second order ordinary differential equations: shooting methods and difference methods. Methods of approximation of boundary value problems for second order ordinary differential equations: Ritz-Galerkin method. Difference methods for first order partial differential equations. CFL condition. Recalling basic facts of theory of second order partial differential equations. Difference approximation of elliptic boundary value problems on the plane. Variational formulation of boundary value problems for elliptic type equations. Ritz-Galerkin and finite element methods for elliptic problems. Difference methods for parabolic problems. Explicit and implicit schemes for heat conduction equation. Stability of approximate method. Cranck-Nicholson scheme for equations of parabolic type. Difference methods for the vibrating string problem and other hyperbolic problems.

Lab: Computer construction of solution of ordinary differential equations. Practical verifying of efficacy of automatic exactness control. Visualization and comparison of usefulness of various methods. Algorithms for numerical methods of solution of one-dimensional boundary value problems for elliptic equations. Discretization of hyperbolic first order problems. Conditions of stability and convergence of approximate methods. Discretization of two-dimensional boundary value problem for elliptic equations. Difference schemes of approximation of one-dimensional parabolic equation. Difference method of discretization of the vibrating string equation.

DIFFUSION PROCESSES ON COMPLEX NETWORKS

Lectures: Introduction to complex networks. Diffusion and random walks. Epidemic spreading in population networks. Rumor and information spreading. Opinion formation processes. Diffusion of innovation.

Lab: Solving problems illustrating the content presented in the lectures.

STATISTICAL PACKAGES

Lectures: Descriptive statistics. Graphical representation of data. Comparison of two populations - Student test, nonparametric tests. Estimation of proportion. Chi-square goodness of fit test. Cross tabulation. Chi-squared test of independence. Simple linear regression - model, estimation, testing. Simple linear regression - prediction, checking assumptions, transformations. Multiple linear regression - estimation, testing, checking assumptions. Multiple linear regression - analysis of variance, coefficient of determination. Multiple linear regression - the sum of the squares, generalized linear tests. Multiple linear regression - correlated predictors, the model selection criteria. Univariate analysis of variance - model, estimation of parameters, testing. Multivariate analysis of variance. Mixed models and generalized linear model.

Lab: Getting familiar with selected statistical package. Descriptive statistics and graphical representation of data. The problem of two samples - Student tests, nonparametric tests, testing normality, graphical representation of data. Tests and confidence intervals for the ratio - the proportion of a single ratio, chi-square goodness of fit test, chi-squared test of independence, graphical representation of data. Simple linear regression - estimation, prediction, power, graphical representation of data and results. Simple linear regression - diagnostics, transformations of variables. Multiple linear regression - estimation, prediction, testing, diagnosis, selection of relevant variables. Analysis of variance - estimation, testing, comparison between groups, diagnostics

QUEUES AND COMMUNICATION NETWORKS

Lectures: Basic concepts from Markov processes theory. An outline of the theory of point processes. Steady state analysis of Markovian queues. Erlang Loss System. Open Jackson network and Gordon-Newel network. Multi-class Queue. Multiserver queus and various queue disciplines. Queues with feedback and loss systems . Transient analysis of Markovian queues .

Classes: Illustration of all models.. Analytical and computer methods. Examples of queuing models.

COMPUTATIONAL FINANCE

Lectures: Derivatives: forwards, futures, swaps and options. Portfolio construction and pricing. Sensitivity analysis. Binomial pricing: CRR, JR and „exact” trees. Hedging strategies. Trinomial trees. Binomial and trinomial pricing of path dependent derivatives. Monte Carlo (MC): Euler and Milstein schemes, variance reduction, correlated variates, quasi-random numbers. MC pricing of American options. Finite difference schemes: explicit, implicit, Crank-Nicolson, hopscotch. Partial differential equations technique.

Lab: Implementation (Matlab, R, Excel/VB, C++, Java and/or Python) of algorithms and methods discussed during lectures.

INTRODUCTION TO BIG DATA ANALYTICS

Lectures: Introduction to Big Data. Big data platforms. Hadoop ecosystem. Querying big data with Hive. Big data and machine learning. In-memory big data platform - Spark. Linked Big Data. Big data visualization.

Project: Practical preparation and presentations of projects illustrating methods given in the lectures.

INTRODUCTION TO APPLIED FLUID DYNAMICS

Lectures: Reminder of the vector analysis elements. Reminder of the complex analysis elements. Conformal mappings. Laws of conservation. Equations of motion for an ideal fluid. Elementary viscous flow. Waves. Shock waves modelling. Classical aerofoil theory. Nonlinear models in diffusion phenomena. Boundary layers. Computational fluid dynamics (CFD).

Project: Preparation and presentations of projects illustrating theory given in the lectures.

MATHEMATICAL IMAGE PROCESSING

Lectures: Overview of fundamental problems in image processing. Representation of images. Models of image degradation. Linear diffusion filter. Gaussian smoothing in the frequency domain. Nonlinear diffusion filters. Isotropic and anisotropic diffusion models. Discretization of the nonlinear diffusion filter. Introduction to variational models for image restoration. Image denoising by total variation regularization. First order numerical schemes for total variation minimization. Image deblurring model. Total variation model for image inpainting. The Mumford-Shah model for image segmentation and its approximations. Active contours model for image segmentation.

Lab: Basic operation on images. Degradation of images. Gaussian smoothing. Solving selected problems illustrating theory given in the lectures using mathematical MATLAB package for numerical computing.

ESTIMATION THEORY

Lectures: Basic concepts of estimation theory: bias, variance, mean square error matrix of Fisher information, efficiency, asymptotic normality. Theoretical basis of simulation methods and replication. Bias and variance estimation - bootstrap, Jacknife, the delta method. Construction of confidence intervals - classic and boostrap intervals. Nonparametric density estimation - histogram and its properties. Nonparametric density estimation - kernel estimator and its properties. Selection of bandwidth in the kernel estimator. Modifications of kernel estimator - variable bandwidth, higher-order kernels. Estimation of density through orthogonal expansions. Estimation of density - local likelihood function and maximum likelihood method with smoothing. Nonparametric regression function estimation - estimation of kernel. Selection of the bandwidth and modification of the kernel estimator of regression function. Hazard function estimation - parametric and nonparametric methods. Empirical Bayesian methods - Stein estimator

Lab: Parametric estimation - method of maximum likelihood. Bias, variance, mean square error - estimation using computer simulations. Estimation of bias, variance and construction of confidence intervals using the method of substitution and replication methods (bootstrap, jackknife). Estimating the quality of estimators based on simulation studies. Estimating the several parameters - asymptotic covariance matrix, the covariance matrix estimation using the method of substitution and replication methods. Estimating the quality of estimators based on simulation studies. Nonparametric estimation of density - the histogram, method of the nearest neighbor, kernel estimator, orthogonal expansions. Smoothing parameter selection. Quality rating based on simulation studies. Nonparametric estimation of the regression function. Estimators: kernel, local polynomial, the nearest neighbor, the smooth spline functions. Construction of confidence intervals and bands using the bootstrap method. Smoothing parameter selection. Quality rating based on simulation studies. Estimation of survival function and hazard function with parametric and nonparametric methods. Construction of confidence intervals through approximation with the normal distribution and the bootstrap method. Quality rating based on simulation studies. Empirical Bayesian methods. Quality assessment using simulation studies.

PERTURBATION METHODS

Lectures: Examples of problems leading to perturbation method. Regular perturbation method. Poincare-Lindstedt method. Asymptotes. Unreliability of the regular perturbation method. Singular perturbation method. The inner and outer approximations. Analysis of shoreline layer. Inner approximation and scaling. Combining internal and external approximation. Uniform approximation. Examples of uniform approximation. Phenomena associated with the film edge. Partial differential equations and perturbation methods. Algebraic equations and perturbation methods.

Lab: Solving problems illustrating a lecture given theory using MATLAB

ANALYSIS OF UNSTRUCTURED DATA

Lectures: Data analysis in Python – PANDAS primer. Retrieving and storing data. Data visualisation. Data wrangling. Natural language processing with NLTK. Sentiment analysis. Document classification. Handling big data.
Project: Practical preparation and presentations of projects illustrating methods given in the lectures

RESERVES IN LIFE AND NON-LIFE INSURANCE

Lectures: Introduction to the course, survey over provision types. Net reserves in life insurance. Decomposition of the loss random variable (Hattendorff's theorem). Technical gain. Gross reserves in life insurance, Zillmer's reserve. Multiple decrement model: net premiums and reserves. Multiple life insurance: net premiums and reserves. Provisions in non-life insurance, including loss data triangles, chainladder method, IBNR, premium reserve. Solvency II - technical provisions, best estimate, risk margin, technical provisions for accounting purposes.

Classes: Solving of problems illustrating theory given in the lectures, solving of problems from actuarial exams.

FREE BOUNDARY PROBLEMS

Lectures: Remaining basic theory of elliptic and parabolic partial differential equations. Stefan problem, notion of the free boundary. Inverse Stefan problem. Free boundary problems in melting and freezing. Modeling of problems connected with phase transition. Modeling of flows in porous media: Boussinesq equation, porous media equation. Self-similar solutions of porous media equation. Free boundary in solutions of porous media equation, finite speed of propagation of disturbances. Retention and penetration property. Large time behavior of solutions. Free boundary in reaction–diffusion–convection equations. Diffusion in solids. Free boundary problems. Modeling of flows in deformable media, spreading of impurities. Free boundary problems in digital image processing. Free boundary problems in financial mathematics. Stationary free boundary problems: dam problem, obstacle problems in calculus of variations.

Classes: Solving of problems illustrating theory given on lectures.

INTRODUCTION TO INVERSE PROBLEMS

Lectures: Introduction to inverse problems. Definition of the well-posedness. Important classes of inverse problems. Differentiation of a noisy data. Computerized tomography. The Radon transform. Inverse problems in image processing. Parameter identification problems. Ill-conditioned matrix equations. Regularization of linear ill-posed problems. Tikhonov regularization. Maximum entropy regularization. Total variation regularization. Estimation of the regularization parameters. Iterative regularization

Lab: Solving problems illustrating the methods given in the lecture using MATLAB package for scientific computing

OPERATIONS RESEARCH

Lectures: Introduction to operations research. Formulation of the linear programming problem. Building mathematical models. The simplex algorithm for linear programming. Duality and sensitivity analysis in linear programming. Algorithms for integer linear programming. Minimum cost flow problem – applications and mathematical properties. Network simplex algorithm. The shortest (longest) path problem – applications and algorithms. The maximum flow problem – applications and algorithms. The assignment, minimum spanning tree and travelling salesperson problems – applications and algorithms. Elements of computational complexity, NP-hard combinatorial optimization problems and limitations of modern computational techniques. Multiobjective programming.

Lab: Introduction to MathProg (AMPL) language. Building and implementing linear programming models for chosen problems. Building and implementing integer linear programming models for chosen problems. Building and implementing models for the minimum cost flow problem and its variants. Building and implementing models for various variants of the travelling salesperson problem. Building and implementation models for chosen combinatorial optimization problems. Building and implementing models for chosen multiobjective problems.

COMPUTER SIMULATIONS OF STOCHASTIC PROCESSES

Lectures: Generation of stable distributions and vectors. Simulation of stable processes by integral and series representations. Self-similar and stationary processes. Generating processes with long memory. Stable models with long memory in physics and economics.

Lab: Solving problems illustrating methods given in the lecture.