Summary of PhD Thesis

“Representations and applications of stochastic processes with resetting”

author: Kacper Taźbierski

The thesis begins by introducing the concept of stochastic resetting, a mechanism that fundamentally alters the behavior of stochastic processes by inducing non-equilibrium steady states and modifying statistical properties like mean first-passage times (MFPT). This phenomenon has wide-ranging applications in physics, biology, ecology, and computer science. The introduction highlights the key feature of resetting: its ability to render infinite MFPTs finite, with a non-monotonic dependence on the resetting rate, which allows for the optimization of random search processes. The thesis aims to contribute to this field by presenting new results on jump-diffusion models, first-passage problems, memory effects, and doubly stochastic mechanisms.

The thesis proposes modeling resets as jump events within stochastic differential equations, leading to jump-diffusion models with resetting guided by a Poisson process that can be analyzed using Itô calculus. This approach is generalized to handle non-Poissonian and time-inhomogeneous resetting.

An alternative representation is introduced, which involves ”gluing” together paths of independent and identically distributed (IID) processes at the renewal times of a resetting process. This framework is particularly useful for analyzing processes with non-Markovian resetting and allows for the derivation of joint two-time probability density functions and autocovariance functions.

Chapter 3 investigates doubly stochastic resetting, where the resetting process itself is governed by another stochastic process, such as a Cox process. This introduces an additional layer of randomness, which captures scenarios in which the reset intensity is subject to environmental fluctuations.

Chapter 4 introduces the concept of memory resetting, where the process, instead of being reset to a fixed point, forgets its history and restarts its evolution. This is distinct from spatial resetting and is modeled by restarting the underlying driving process.

Chapter 5 is dedicated to the analysis of first-passage times under resetting. Stochastic resetting is shown to be a mechanism that can render otherwise infinite mean first-passage times finite and optimizable. The thesis provides a general formula for the MFPT under a renewal-based resetting protocol. Furthermore, the research covers interesting results regarding first-passage times (FPTs) for processes with drift.

Chapter 6 extends the classical arcsine laws for Brownian motion to the case with Poissonian resetting. The distributions of the time spent on the positive half-line and the time of the last zero-crossing are all significantly altered by the resetting mechanism.