Streszczenie rozprawy doktorskiej

pt. „Discrete Feynman-Kac evolutions" w języku angielskim

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We develop the theory of discrete-time Feynman–Kac semigroups with general confining potentials for Markov chains defined on countably infinite discrete spaces. We concentrate on chains exhibiting a long-range distributional feature known as the direct step property (DSP), which asserts that the two-step transition probability is, up to a constant factor, dominated by the one-step transition probability. For context and comparison, we also consider, in part, the classical case of nearest-neighbour random walks on graphs with finite geometry.

We first establish that the DSP holds for a broad class of Markov chains, including stable-like random walks and kernels with lighter tails, and show that it remains stable under random time-changes, allowing for further constructions via discrete subordination.

We then derive sharp two-sided estimates for harmonic functions of Feynman–Kac operators in the DSP setting, yielding a discrete version of the uniform Boundary Harnack Inequality at infinity. A comparison with nearest-neighbour random walks reveals notable contrasts, and these estimates are applied to both graph Laplacians with confining potentials and eigenfunctions of Feynman–Kac semigroups.

We further study discrete-time Feynman–Kac semigroups and their duals in the non-self-adjoint setting. Sharp two-sided kernel estimates are derived in terms of the ground state, yielding the discrete-time analogue of progressive intrinsic ultracontractivity (pIUC) and its stronger asymptotic form (aIUC). Additionally, aIUC is shown to be equivalent to intrinsic hypercontractivity, establishing a direct link between the growth of the potential and the smoothing properties of the semigroup. Interestingly, in the case of nearest-neighbour random walks on graphs of finite geometry, typical examples fail to satisfy either aIUC or IHC, highlighting a fundamental difference between long-range and nearest-neighbour dynamics.

The long-time behaviour of these semigroups is also investigated. We prove ergodicity of the intrinsic semigroup and quasi-ergodicity of the original one in l^p-spaces, with particularly delicate results in l^1, where new equivalences between these properties are established. In particular, these results build on the pIUC property, which provides explicit convergence rates and ensures progressive in-time uniform (quasi-)ergodicity of the semigroups. In the stronger aIUC regime, both semigroups are uniformly (quasi-)ergodic with geometric rates.

The dissertation is closed by illustrating the theory with concrete examples showing how the interplay between transition kernels and confining potentials influences contractivity and ergodicity.