WROCŁAW UNIVERSITY OF TECHNOLOGY – PHD STUDIES

FACULTY OF PURE AND APPLIED MATHEMATICS

SUBJECT CARD

Course name in Polish: Układy Dynamiczne i Teoria Ergodyczna

Course name in English: Dynamical Systems and Ergodic Theory

Course language: English

University-wide general course type:
1) basic course (mathematics, physics, chemistry, other)
2) humanity course
3) managerial skills
4) English language
5) other modern language

Departmental course developing professional skills:
1) specialized course
2) interdisciplinary course
3) seminar (interdisciplinary, specialized, departmental)

Type of course (obligatory, optional)

Educational effects according to ZW 26/2017 regulations:
P8S_WG, **P8S_UW**, **P8S_KK**, **P8S_KR**

Subject code: MAT1306

*delete as applicable

	Lecture
Number of hours of organized classes in University (ZZU)	30
Number of hours of total student workload (CNPS)	90
Form of crediting	Exam
Number of ECTS points	3
including number of ECTS points for practical (P) classes	
including number of ECTS points for direct teacher-student contact (BK) classes	2

PREREQUISITES RELATING TO KNOWLEDGE, SKILLS AND OTHER COMPETENCES

1. Basic knowledge in the areas of Topology, Measure Theory and Functional Analysis.
2. Ability to study supplementary areas of knowledge and skills.

SUBJECT OBJECTIVES	
C1	The student will learn selected notions of the Theory of Dynamical Systems including Ergodic Theory, and main theorems concerning these notions.
C2	The student will be able to recognize the connections between the notions of the Theory of Dynamical Systems and notions from other areas of mathematics, such as Stochastic Processes, Differential Equations, or Number Theory.
C3	The student will acquire the skills of applying selected tools of the Theory of Dynamical Systems to solving problems, including problems from other areas of mathematics and physics.

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SUBJECT EDUCATIONAL EFFECTS
Relating to knowledge: PEK_W01 – Student knows basic methods of the Theory of Dynamical Systems PEK_W02 – Student knows the connections with and applications of the above notions to other areas of mathematics **Relating to skills:** PEK_U01 – Student acquires skills and ability connected to the scientific methodology of research PEK_U02 – Student acquires skills and ability in creating and self-sufficient conduct of scientific research **Relating to social competences:** PEK_K01 – Student is aware of the role of cooperation, including an international cooperation. PEK_K02 – Student is aware of the importance of the original research activity

PROGRAM CONTENTS		
Form of classes – lecture		Number of hours
Lec 1	Abstract notion of a dynamical system with the action of a group or subgroup, examples	2
Lec 2	Topological dynamical systems, transitivity, minimality, recurrence, factors, conjugacy, proximal pairs, asymptotic pairs, Li-Yorke chaos	2
Lec 3	Symbolic systems. Properties, characterization, applications	2
Lec 4	Connections with Number Theory – IP sets and other. Mention of Sarnak Conjecture	2
Lec5	Measure-preserving transformations. Factors and isomorphism. Recalling the basic theorems: Poincare recurrence, Rokhlin Lemma and Ergodic Birkhoff Theorem. Connections with stochastic processes.	2
Lec6	Invariant measures in a topological system. Simplex of measures. Mention of the Furstenberg Conjecture.	2
Lec7	Spectral properties of a dynamical system. Koopman Operator and Markov Operator. Spectral Theorem. Point Spectrum, pure point spectrum (measure theoretic and topological). Equicontinuity. Maximal equicontinuous factor, Kronecker factor. Group rotation. Halmos – von Neumann Theorem.	4
Lec8	Mixing, weak mixing, Bernoulli systems.	2
Lec9	Various constructions of dynamical systems: induced maps, suspensions, skew products, group extensions, semicocycle extensions, special flows, elements of the theory of Smoothe Systems (Smale horseshoe).	6
Lec10	Topological and measurable joinings. Disjointness	2
Lec11	Topological and measure-theoretic entropy. Conditional entropy. Subadditivity.	2
Lec12	Applications of entropy, Variational principle, Ornstein Theorem.	2
	Total hours	30

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TEACHING TOOLS USED	
N1	lecture
N2	consultations
N3	written assignments: problem solutions

EVALUATION OF ACHIEVED SUBJECT EDUCATIONAL EFFECTS		
Evaluation: F – forming (partial) C – concluding	Educational effect number	Way of evaluating achievement of educational effects
F1	PEK_W01, PEK_W02, PEK_U01, PEK_U02,	participation in the course
F2	PEK_W01, PEK_W02, PEK_U01, PEK_U02,	solutions of the problems
C = 0. 5*F1 + 0.5*F2		

PRIMARY AND SECONDARY LITERATURE

PRIMARY LITERATURE:

[1] Peter Walters: An Introduction to Ergodic Theory, Springer 1982
[2] Karl Petersen: Ergodic Theory
[3] J. de Vries: Elements of Topological Dynamics

SECONDARY LITERATURE:

[1] T. Downarowicz: Entropy in Dynamical Systems , Cambridge University Press 2011

SUBJECT SUPERVISOR
(NAME AND SURNAME, E-MAIL ADDRESS)

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MATRIX OF CORRELATION BETWEEN EDUCATIONAL EFFECTS FOR SUBJECT DYNAMICAL SYSTEMS AND ERGODIC THEORY AND EDUCATIONAL EFFECTS FOR MAIN FIELD OF STUDY

Doctoral studies at Faculty of Pure and Applied Mathematics

Subject educational effect	Correlation between subject educational effect and educational effects defined for main field of study and specialization (if applicable)**	Subject objectives***	Programme content***	Teaching tool number***
(knowledge) PEK_W01	**P8S_WG**	C1, C2	Lec1-15	N1, N2, N3
PEK_W02	**P8S_WG**	C1, C2	Lec1-15	N1, N2, N3
(skills) PEK_U01	**P8S_UW**	C2	Lec1-15	N2, N3
PEK_U02	**P8S_UW**	C2, C3	Lec1-15	N2, N3
(competences) PEK_K01	**P8S_KK**	C3	Lec1-15	N2, N3
PEK_K02	**P8S_KR**	C3	Lec1-15	N2, N3

** - enter symbols for main-field-of-study/specialization educational effects

*** - from table above