

# Summary of doctoral dissertation

## "Monitoring of anomalous diffusion processes using time-averaged statistics"

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The thesis addresses the problem of analyzing anomalous diffusion processes using time-averaged statistics. The main focus is placed on parameter estimation, statistical testing procedures, and identification methods. Various models of anomalous diffusion, widely discussed in the literature, are considered. To construct methodologies for parameter estimation, testing, and identification, we rely on classical time-averaged statistics and introduce new ones, namely empirical anomaly measure (EAM), even empirical moments (EEM), and time-averaged mean squared displacement ratio (TAMSD ratio). The new methodologies allow for improved accuracy and robustness in the analysis of anomalous diffusion.

In the dissertation, various problems related to anomalous diffusion are discussed. The focus is placed on parameter estimation and statistical testing of Gaussian processes. The study also addresses a gap in the literature by developing methodologies for testing two-dimensional processes. Furthermore, new approaches are proposed to distinguish between Gaussian processes and those with distributions close to Gaussian. Particular attention is given to models with random parameters, which constitute a new class of processes within this field. Since anomalous diffusion phenomena are observed in a wide range of disciplines, including physics, biology, hydrology, climate science, medicine, and finance, the developed methodology is illustrated with several applications. Theoretical results are supported by Monte Carlo simulations and validated with real-world datasets.

The thesis is structured as follows. Chapter 1 is the introduction to the problem and provides research questions and hypotheses. Chapter 2 presents the state of the art on anomalous diffusion processes, including definitions of the processes, parameter estimation methods, and approaches to testing. In addition, the original contributions of the author to this field are outlined, and scientific achievements are listed.

Chapter 3 provides the definition and properties of the considered anomalous diffusion processes, namely fractional Brownian motion (FBM), two-dimensional FBM, fractional Brownian motion with random Hurst exponent (FBMRE), fractional Lévy stable motion (FLM), scaled Brownian motion (SBM), scaled Brownian motion with random anomalous diffusion exponent (SBMRE), Gaussian Ornstein-Uhlenbeck process (GOU), and Lévy Ornstein-Uhlenbeck process (LOU). In the following sections, these processes are used to illustrate the properties of time-averaged statistics and to evaluate the performance of new methodologies based on them.

In Chapter 4, we provide definitions and summarize the properties of both classical and newly proposed time-averaged statistics. The classical statistics include time-averaged mean squared displacement (TAMSD), sample autocovariance function (ACVF), detrended fluctuation analysis (DFA), detrended moving average (DMA), and sample cross-covariance function (CCF). The newly introduced statistics are empirical anomaly measure (EAM), even empirical moments (EEM), and TAMSD ratio.

Chapter 5 investigates the performance of time-averaged statistics, including TAMSD, DFA, DMA, and ACVF, in estimating the Hurst exponent for FBM and FBM with ad-

ditive noise. Through Monte Carlo simulations, we assess and compare the accuracy and characteristics of these estimation methods for the specified stochastic processes.

Chapter 6 focuses on testing Gaussian processes. In the first section, the novel algorithms based on the EAM statistic are presented. In addition, the comparison with the relevant ACVF-based method is provided. In the next section of this chapter, we address the problem of eliminating the influence of the diffusion coefficient in the testing procedure. We demonstrate that using the TAMSD ratio statistic yields more accurate results compared to employing the classical TAMSD statistic in the testing process. In the last part of this chapter, the novel methodology based on CCF statistic for two-dimensional FBM is presented. The effectiveness of all methodologies is examined using both Monte Carlo simulations and real-world datasets, including a biological dataset and a dataset of Bitcoin prices.

In Chapter 7, we focus on the fact that, in some cases, processes with fixed parameters may be inadequate to describe real complex data. To address this problem, we concentrate on anomalous diffusion processes with random parameters, namely FBMRE and SBMRE. It is shown that, for certain datasets, these models provide a better fit. Based on this observation, we propose new methods to distinguish between models with fixed and random parameters. We construct new methods based on six statistics, namely TAMSD, EAM, DMA, and their ratio counterparts. The effectiveness of the proposed methodology is illustrated using both a simulation study and a biological dataset.

In Chapter 8, the discrimination methodology between models with finite and infinite moments is presented. This new approach, based on EEM statistic allow us to distinguish between Gaussian processes and processes of distributions close to a Gaussian one. Based on Monte Carlo simulations and real-world data (the closing price of Copper (CU) in USD), we demonstrate that the approach based on the EEM statistic performs significantly better than classical methods.

In Chapter 9, the thesis is concluded by summarizing the presented work and discussing the research hypotheses formulated at the beginning of the study.

The results of the dissertation were described in eight scientific articles.

