

WROCLAW UNIVERSITY OF TECHNOLOGY – PHD STUDIES

FACULTY OF PURE AND APPLIED MATHEMATICS
SUBJECT CARD
Course name in Polish: Wybrane Zagadnienia Analizy Funkcjonalnej
Course name in English: Selected Problems of Functional Analysis
Course language: English
University-wide general course type: 1) <u>basic course</u> (mathematics, physics, chemistry, other) 2) humanity course 3) managerial skills 4) English language 5) other modern language Departmental course developing professional skills: 1) specialized course 2) interdisciplinary course 3) seminar (interdisciplinary, specialized, departmental)
Type of course (obligatory, <u>optional</u>)
Educational effects according to ZW 26/2017 regulations: P8S_WG, P8S_UW, P8S_KK, P8S_KR
Subject code: MAT1307

*delete as applicable

	Lecture
Number of hours of organized classes in University (ZZU)	30
Number of hours of total student workload (CNPS)	90
Form of crediting	Exam
Number of ECTS points	3
including number of ECTS points for practical (P) classes	
including number of ECTS points for direct teacher-student contact (BK) classes	2

PREREQUISITES RELATING TO KNOWLEDGE, SKILLS AND OTHER COMPETENCES

1. Student is able to handle methods of functional analysis.
2. Student is able to study supplementary areas of knowledge and skills.

SUBJECT OBJECTIVES

C1 Student will learn selected notions of advanced functional analysis, including notions of unbounded operators, distributions and Sobolev spaces, as well as their applications in the theory of partial differential equations and quantum mechanics

C2 Student will acquire the skills to use selected concepts of functional analysis in his/her research in mathematics and/or theoretical physics

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SUBJECT EDUCATIONAL EFFECTS

Relating to knowledge:

PEK_W01 – Student has knowledge of advanced methods of functional analysis

PEK_W02 – Student has knowledge of applications of the methods of functional analysis

Relating to skills:

PEK_U01 – Student gains skills needed to perform his/her research.

PEK_U02 – Student is able to conduct his/her research.

Relating to social competences:

PEK_K01 – Student is aware of the role of cooperation, including an international cooperation.

PEK_K02 – Student is aware of the importance of the original research activity

PROGRAM CONTENTS

Form of classes – lecture		Number of hours
Lec 1	Unbounded operators: definition, operator extension, graph, closability, spectrum, examples.	2
Lec 2	Self-adjoint operators: adjointness, symmetry, self-adjointness and essential self-adjointness, criteria for self-adjointness	2
Lec 3	Spectral theorem for bounded hermitian operators: functional calculus, spectral projectors and measures.	2
Lec 4	Spectral theorem for unbounded self-adjoint operators: three forms of the spectral theorem.	2
Lec5	Illustration of the spectral theorem: Laplacian and harmonic oscillator.	2
Lec6	Mathematical model of quantum mechanics.	2
Lec7	Semigroups of operators: heat and Poisson semigroups for positive self-adjoint operators with illustration for Laplacian and harmonic oscillator, one-dimensional unitary groups of operators, Stone's theorem.	2
Lec8	Introduction to the distribution theory: basic ideas, weak derivatives, test functions.	2
Lec9	Elements of Fourier analysis: Schwartz' space, convolutions.	2
Lec10	Distributions: operations on distributions, convergence in the spaces of distributions, different kinds of distributions.	2
Lec11	Tempered distributions: Fourier transform of a distribution.	2
Lec12	Applications to main equations of mathematical physics: fundamental solutions, examples of fundamental solutions for classic differential operators.	2
Lec13	Sobolev spaces H_s : definition, basic properties, Sobolev embedding theorem.	2
Lec14	Local Sobolev spaces $H_{s,loc}$: definition and properties, elliptic operators, elliptic regularity theorem.	2
Lec15	Sobolev spaces $W^{k,p}$: basic properties.	2
Total hours		30

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TEACHING TOOLS USED	
N1	lecture
N2	consultations
N3	written assignments: problem solutions

EVALUATION OF ACHIEVED SUBJECT EDUCATIONAL EFFECTS		
Evaluation: F – forming (partial) C – concluding	Educational effect number	Way of evaluating achievement of educational effects
F1	PEK_U01, PEK_U02, PEK_W01, PEK_W02	participation in the course
F2	PEK_W01, PEK_W02, PEK_U01, PEK_U02, PEK_K01, PEK_K02	solutions of the problems
$C = 0.5 \cdot F1 + 0.5 \cdot F2$		

PRIMARY AND SECONDARY LITERATURE
<u>PRIMARY LITERATURE:</u> [1] J. Conway, A Course in Functional Analysis, Graduate Texts in Mathematics, Springer, 1997 [2] M. Reed and B. Simon, "Methods of Modern Mathematical Physics", vols. 1-4, Academic Press, 1972 [3] W. Rudin, Analiza funkcjonalna, PWN, 2001 <u>SECONDARY LITERATURE:</u> [4] R. Strichartz, A Guide to Distribution Theory and Fourier Transforms, World Scientific 1994 [5] S. Ramamurti, Mechanika kwantowa, PWN, 2007

SUBJECT SUPERVISOR (NAME AND SURNAME, E-MAIL ADDRESS)
prof. dr hab. Krzysztof Stempak, Krzysztof.Stempak@pwr.edu.pl

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**MATRIX OF CORRELATION BETWEEN EDUCATIONAL EFFECTS
FOR SUBJECT
SELECTED PROBLEMS OF FUNCTIONAL ANALYSIS
AND EDUCATIONAL EFFECTS FOR MAIN FIELD OF STUDY
Doctoral studies at Faculty of Pure and Applied Mathematics**

Subject educational effect	Correlation between subject educational effect and educational effects defined for main field of study and specialization (if applicable)**	Subject objectives***	Programme content***	Teaching tool number***
(knowledge) PEK_W01	P8S_WG	C1, C2	Lec1-15	N1, N2, N3
PEK_W02	P8S_WG	C1, C2	Lec1-15	N2, N3
(skills) PEK_U01	P8S_UW	C2	Lec1-15	N1, N2, N3
PEK_U02	P8S_UW	C2	Lec1-15	N2, N3
(competences) PEK_K01	P8S_KK	C1, C2	Lec1-15	N2, N3
PEK_K02	P8S_KR	C1, C2	Lec1-15	N2, N3

** - enter symbols for main-field-of-study/specialization educational effects

*** - from table above