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## **Report on the PhD thesis “Properties of Lévy processes in smooth domain” by Tomasz Juszczyszyn**

The PhD thesis under report consists of three papers, which have already been published in top quality journals.

The first paper is entitled “Hitting times of points for symmetric Lévy processes with completely monotone jumps” jointly with Mateusz Kwaśnicki, which was published in *Electronic Journal of Probability*. This work studies one-dimensional symmetric Lévy processes with completely monotone jumps and aims to obtain asymptotic estimates for the derivatives of the probability densities of the hitting times of points. In order to develop the analysis of the derivatives, Mr. Juszczyszyn assumes the scaling-type conditions and succeeds in deduction of precise asymptotic estimates of the derivatives.

The second paper is entitled “Martin kernels for Markov processes with jumps” jointly with Mateusz Kwaśnicki, which was published in *Potential Analysis*. This work studies a class of Markov processes with jumps satisfying certain regularity conditions, which is so general as to contain several examples of Lévy processes. It aims to identify the Martin boundaries of the process considered. Mr. Juszczyszyn obtains the boundary limits of the ratios of harmonic functions in relation to the Harnack inequalities and finally determines the desired Martin boundaries.

The third paper is entitled “Decay rate of harmonic functions for non-symmetric strictly  $\alpha$ -stable Lévy processes”, which was published in *Studia Mathematica*. This work studies multi-dimensional strictly stable Lévy processes and aims



to determine the boundary decay rates of harmonic functions on bounded Lipschitz domains. Utilizing the Ikeda—Watanabe formula involving the Dynkin generator of the process considered, Mr. Juszczyszyn accumulates a series of estimates on the Dynkin generator and finally achieves the proof of the boundary limits as desired.

The PhD thesis is well written. Proofs are worked out in detail and arguments are clear. The main results as described above are impressive new contributions. I strongly recommend to accept this as a PhD thesis.

The PhD thesis of Mr. Juszczyszyn gives original solutions to several important scientific problems. I am absolutely confident that he has excellent theoretical knowledge in the field of probability theory as well as the capacity to carry out independent research.

With best regards,  
Yours sincerely,

*Koji Yano*

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