



THE UNIVERSITY OF
**WESTERN
AUSTRALIA**

Perth, 22 August 2024

The thesis entitled "*Hardy inequalities and nonlocal geometry*" authored by Julia Anna Lenczewska provides an original approach to classical, and important, concepts, such as Hardy inequalities, capacities, heat contents, and curvatures, from the perspective of nonlocal operators.

In Chapter 3 of this dissertation, a Hardy inequality for nonlinear operators in the full Euclidean space is proved and its optimal constant is determined. The method of proof relies on the semigroup theory and produces, as a consequence, a result about the contractivity of the corresponding semigroup. This is a useful and interesting result, which has been published on the renowned Journal of Functional Analysis.

Chapter 4 deals with the case of the half-space and of convex domains. The results presented here are also original and significant.

Chapters 5, 6, and 7 consider a nonlocal notion of capacity induced by a general Lévy measure and a related notion of nonlocal perimeter and heat content. Several new results are presented, including higher-order asymptotics, which are also new for the case of the fractional Laplacian, but also apply to a large class of unbounded operators.

The case of general nonlocal mean curvatures is also considered, in terms of asymptotics, as well as existence, uniqueness, and stability of viscosity solutions.

The thesis, in my opinion, is of excellent quality. The number of results obtained is also impressive. The thesis is also written with care, and it is a pleasure to read it. I recommend this thesis in the highest possible form.

In summary, my assessment is that the general theoretical knowledge of the candidate in mathematics is remarkable, with a great attitude in conducting original scientific work. The dissertation provides many original results about very interesting problems, and I propose it for distinction, given the quality and quantity of the results obtained.

If any additional information is needed, please do not hesitate to contact me.

Kindest regards,

Serena Dipierro

A handwritten signature in black ink that reads "Serena Dipierro". The signature is written in a cursive style with a large initial 'S' and a distinct 'D'.