REPORT: ASPECTS OF DISCRETE HARMONIC ANALYSIS BY WOJCIECH SŁOMIAN

This thesis addresses some questions at the forefront of discrete harmonic analysis. In particular it obtains the most robust - in terms of being uniform with respect to many of the underlying parameters - estimates on the so-called oscillation semi-norms of discrete Radon type operators associated to polynomial mappings.

The first half the thesis gives a very thorough development of the underlying tools, both from discrete harmonic analysis as well as from analytic number theory. This part already demonstrates that the author has the mastery of the relevant areas of research in mathematics, in fact shows a deep understanding of these areas.

The second part of the thesis describes the proofs of the main results of the thesis. The results are original albeit highly technical. They are a culmination of past results toward obtaining stronger more robust oscillation or variational estimates for Radon type operators. As such the thesis resolves important questions in discrete harmonic analysis.

The third part of the thesis is devoted to bootstrap arguments. It starts with a very nice overview of proofs along these lines of some classical results. It then gives new proofs based on bootstrap arguments of some important recent results in discrete harmonic analysis.

One of the most crucial result of the thesis, Theorem 1.45 is claimed to be done entirely by the author (it appeared in a joint paper), which evidences that the author is able to conduct independent scientific work.

Overall, this is a high quality thesis displaying both originality and a mastery of many modern tools in harmonic analysis.

The exposition is well-structured and well-written. The proofs of the theorems and lemmas are given in sufficient detail.

Comments on the exposition.

There are some grammatical and minor mathematical misprints. Without trying to list all of them I'd list such misprints at a few places below, and would recommend another careful proofreading of the thesis.

- *Estimate (1.29) was obtained by Hua not by Hardy and Littlewood* (as noted correctly later)

- *Some error terms seem to be imprecise and/or incorrect on page 19, e.g.*

(i) Should $|\xi - a/q| \leq X_N^{-k+\alpha}$ instead of $X_N^{k-\alpha}$ on the major arcs?

(ii) What does it mean that the error term is $O(N^\delta)$ for some $\delta > 0$?

(iii) For the main term $M_k(N)$, are the error terms $O(X_N^{d-1-\delta'})$ or $O(X_N^{d-k-\delta'})$?

- *In definition 2.9 it seems that the 2-variational seminorm is given instead of the r-variational seminorm*

- *On page 42 there a few confusing formulas, e.g.*

(i) In formula (2.56) is it $\mathcal{P}(y)$ or $(y)^\Gamma$?

(ii) In formulas (2.61)-(2.62) is it $(t)^\Gamma$ or $(y)^\Gamma$?

- *Remarks on pages 44-46,*

(i) On page 44, "In 2002 Magyar the authors..." the name"Magyar" may be omitted.

(ii) On page 45, The meaning of sentence "The second thing is the limitation the underlying" is not clear, may be rephrased.

(iii) On page 46, the expression "the last progression" may be rephrased to "the latest progress".

- *On page 48 there is some confusing notation,*

(i) What is J in formula (3.3)?

(ii) It seems that the parameter N have two different roles in the formula below (3.3)

(iii) In the next formula may be a normalizing factor is missing?

(iv) In Theorem 3.4 in formulas (3.5)-(3.6) the polynomial $\mathcal{P}$ do not appear, i.e. these formulas involve only canonical polynomial, why does $\mathcal{P}$ appear in the statement?

- *General remark on the proofs of Theorem 3.4 and Theorem 3.7*

It would be helpful for the reader to identify the main ideas of the proofs of the uniform oscillation estimates if the proof would be given, or at least sketched, in a model case, for example, for the mapping $\mathcal{P}(y) = y^2$ or $\mathcal{P}(y) = (y, y^2)$ in $l^2(\mathbb{Z})$ i.e. when $p = 2$. The proofs are given in the greatest generality when the heavy notation is making it somewhat hard to separate the technical difficulties from the crucial new ideas.

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