

# DIOSCURI

CENTRE IN TOPOLOGICAL DATA ANALYSIS



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**Review of the Doctoral Dissertation**  
**“Cyclostationary Processes with Additive Noise - Finite - and Infinite-Variance Case”**  
**by mgr inż. Wojciech Żuławiński**

In my opinion the presented thesis is an original solution to a practically important scientific problem and **meets all the requirements stated in Article 187 of the Act of 20 July 2018 “Prawo o szkolnictwie wyższym i nauce”** (as published in Dz. U. 2023, position 742, with later amendments) for the discipline of mathematics. The thesis reflects the candidate's broad theoretical knowledge and ability to conduct scientific research. Below, I provide the details of my assessment of the thesis of mgr inż. Wojciech Żuławiński.

The dissertation presented by mgr inż. Wojciech Żuławiński addresses the field of stochastic processes: cyclostationary processes with additive and multiplicative noise in the case of finite and infinite variance of the noise. These processes, characterized by their periodic and stochastic properties (of time series or their higher moments), are highly relevant for applications in diverse domains, including condition monitoring, communications, finance and more. The data registered by the practitioner in those domains are very often corrupted with a substantial amount of noise and therefore there is a need for robust tools to analyse them. The thesis explores both finite-variance and infinite-variance cases, making theoretical advancements and proposing practical solutions of real-world problems. The presented

methods are both theoretically justified and tested in a number of practical scenarios. In particular, the presented research aims to bridge gaps in the existing literature for cyclostationary processes influenced by additive noise and impulsive behavior. The presented techniques are used to (1) analyze cyclostationary processes with additive noise, (2) detect and quantify the presence and the type of noise, (3) identify cyclostationary behavior under impulsive noise and (4) enhance classical algorithms for practical data analysis.

The thesis is structured as follows; Chapter 1 introduces the problem and hypotheses related to cyclostationary processes with additive noise, focusing on finite and infinite variance cases. Chapter 2 reviews the state of the art and lists the author's significant contributions to the field. Chapters 3 and 4 delve into the classical and advanced methodology of cyclostationary processes in finite and infinite variance cases. These chapters outline key concepts, definitions, and methodologies, such as alpha-stable distributions and novel dependence measures. Chapters 5 through 8 present the Author's work on periodic autoregressive (PAR) models with additive noise, detailing novel estimation methods and validation through Monte Carlo simulations. This includes tailored solutions for both finite and infinite variance cases. The final chapters shift focus to general methodologies and applications not tied to cyclostationary processes. Chapter 9 proposes robust methods for periodicity detection in cyclostationary data, adapted for impulsive noise scenarios. These include frequency domain-based tools such as robust cyclic spectral coherence and new automated period estimation algorithms. Chapter 10 validates these methodologies using simulations and real-world datasets, showcasing their efficacy over traditional approaches. Chapter 11 concludes with a summary of findings and emphasizes their practical relevance, especially in practical tasks of condition monitoring. The appendix extends the discussion by detailing additional heavy-tailed distributions, enhancing the thesis's comprehensiveness.

The work presented in the thesis was published in 8 papers co-authored by the applicant. The applicant, during his phd studies coauthored in total 15 peer-reviewed papers (hence, the results from additional 7 papers are not discussed in details in the thesis). These publications appeared in high-impact journals and prominent conferences, underscoring the quality and relevance of the research. I must say that this result is highly impressive - both in quality and quantity of the research outputs.

The thesis presents several significant contributions, which can be categorized into theoretical advancements, new methodologies, and solutions to practical problems. In terms of theoretical advancements, the dissertation introduces a novel framework for analyzing cyclostationary processes with additive noise in both finite and infinite variance scenarios. The author developed new estimation methods, modified Yule-Walker equations, and robust periodicity detection techniques tailored for impulsive data. For infinite variance processes, the thesis extends classical dependence measures to address the challenges posed by heavy-tailed distributions. Regarding new methodologies, the thesis presents a suite of robust estimation algorithms for model parameters (including PAR), noise testing, and periodicity detection. Additionally, it introduces advanced spectral analysis techniques, such as robust cyclic spectral coherence and measures for automated period estimation. In terms of practical applications, the proposed methods are thoroughly validated on real-world datasets from the condition monitoring domain.

The thesis is well-structured, beginning with a comprehensive review of the state of the art and progressing systematically through theoretical contributions, methodology, and applications. Each chapter builds on the previous one, culminating in robust conclusions that align with the stated research goals. I would suggest advancing the notation in the algorithms - in particular

specifying what the input and the output is. Most notably, I was not able to find links to the implementation of the presented algorithms, although they are sketched in detail in the thesis and results of the implementation are broadly discussed. While the implementation is conceptually simple to do, I believe providing even a basic library in mathematica or python would increase the scope of the potential uses of the techniques by far - in particular since the prototype implementation was already used in the extensive tests of the methods. I also believe that a PhD thesis should have an abstract in Polish, which the following thesis is lacking. Those are however minor problems. To my knowledge, the dissertation introduces models and methods that have not been previously known in the literature. Proposed techniques are not only properly theoretically justified by the Author, but are demonstrated to be effective in real-world scenarios, particularly in the condition monitoring of mechanical systems via extensive Monte Carlo simulations and real data tests. To my best understanding the mathematics behind the results is correct and they are used to solve important practical problem. The research is conducted in a modern environment of broad projects, such as NonGauMech. The emphasis on interdisciplinary and practical relevance is noticeable.

It should be also noted that during his doctoral studies, mgr Żuławiński received a number of awards and scholarships. These include a Rector's Award for scientific achievements in 2021/2022, the second prize for the best young scientist presentation at the XLVIII Conference on Mathematical Statistics in 2022, scholarship from WUST's as well as the Quintus Program Award in 2023 for teaching excellence. This recognition indicates a very strong presenting and teaching style of mgr Żuławiński which is an additional added value to the portfolio of this promising young scientists.

### **Summary.**

After reviewing the doctoral dissertation of mgr inż. Wojciech Żuławiński entitled "Cyclostationary Processes with Additive Noise - Finite - and Infinite - Variance Case," I am convinced that it meets the requirements set forth for doctoral dissertations by the relevant academic regulations. I believe that the dissertation provides an original solution to a scientific problem and contains innovative mathematical methods alongside valuable scientific results. The presented work demonstrates that the author possesses the comprehensive theoretical knowledge expected of a doctoral degree holder in the field of exact and natural sciences, specifically in the discipline of mathematics, and has mastered the research methods necessary to conduct advanced scientific work. I therefore **recommend admitting mgr inż. Wojciech Żuławiński to the next stages of the procedure for awarding the PhD degree in the discipline of mathematics.** Moreover, **I propose this doctoral dissertation be considered for distinction**, as the results achieved hold significant scientific importance, and the dissertation has been written with thoroughness and precision.