Referee report on Cyclostationary processes with additive noise – finite- and infinite-variance case (PhD thesis)

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The thesis deals non-stationary processes. Modeling of such processes is rather difficult and there is no unified theory nor approach. The candidate deals with a version of so-called *cyclostationary* processes, a class of processes where the periodicity does not manifest itself directly in a process trajectory, but is present in e.g. its mean and/or in its dependence structure. In particular, cyclostationary processes with heavy tails are of interest. The topic seems to be present primarily in an applied literature, with limited theoretical contribution (with few exceptions, such as the paper by Anderson and Meerschaert in *Annals of Statistics* in 1997). As such, the author's contribution is very important and very welcomed. In fact, some of the results established in the thesis have been published already.

I find this thesis to be very nicely and carefully written. It is a very nice methodological thesis, yet with limited theoretical development.

1 Scholarly Merit

1.1 Structure and contents of the thesis

Chapter 2 includes an extended literature review along with a detailed description of the author's contribution.

Chapter 3 defines strictly and weakly cyclostationary processes.

Chapter 4 introduced cyclostationary processes with infinite variance. For this, stable distribution is recalled along with the dependence measures suitable for processes with infinite variance. Instead of covariance, the normalized covariation (NCV) is considered (see Definition 4.2.2 and Eq. (4.25)). A cyclostationarity is defined through a periodic behaviour of the NCV - see Definition 4.3.1.

Chapters 5 and 6 deal with cyclostationary time series with additive noise. They are built from a periodic ARMA model along with an additive white noise process (see Definition 5.2.1). Theory, methodology and estimation is a relatively straightforward extension of the corresponding results for stationary time series, but certainly an implementation requires some care. In particular, Section 5.3.4 on constrained least squares is quite technical.

Chapters 7 and 8 deals with cyclostationary time series with infinite variance. The novelty, I believe, is the Yule-Walker-type estimation procedure, based on NCV. I think this novelty should be emphasized much clearer.

Chapters 9 and 10 present novel frequency-domain methods to analyse cyclostationary processes. It starts with a nice example that illustrates the need to consider a robust version of ACF, that is not affected by outliers. This leads to robust Fourier transform (spectral version of ACF), robust spectral coherence and robust coherent statistics. My problem with the latter, as indicated above, is that I do not know what they are suppose to estimate. Chapter 10 includes a lot of data analysis.

2 Comments

Major comments:

- In Definition 3.3.1, the author recalls the sample coherence. What does it suppose to estimate? What does it estimate if we have a strictly stationary process?
- I think for the reader who is very familiar with stationary time series, but is not extremely familiar with cyclostationary processes, some links between these two classes of processes would be useful. For example, it would be useful to show a trajectory of a cyclostationary process along with a plot of ACVF to illustrate the associated periodicity (for instance, in the context of Example 3.4.2 or 4.5.2). As such, I would welcome a plot of sample ACVF additionally to Figure 3-4 or a plot of sample NCV additionally to Figure 4-4.
- I feel that some data analysis is missing. In the simulated examples (such as periodic AR in Chapter 4) the sample ACF shows a very nice, periodic behaviour. I presume, when dealing with real data, such periodic behaviour is not prominent. in this context, does it make sense to test hypothesis of stationarity vs. periodicity?

- In Chapter 7, the author proposes Yule-Walker-type estimators, based on NCV measure. I feel personally that such estimators should be accompanied by a large sample theory, e.g. to construct confidence intervals. Is anything known in this direction.

- On page 107, the author writes: "Let us note that, in this chapter, the Yule-Walker equations are based on NCV and FLOC measures."

  On the other hand, in case of finite variance, we can use Yule-Walker equations based on ACF and its sample version to define Yule-Walker estimators. In the infinite variance case, ACF is not defined but we can still consider sample ACF and hence Yule-Walker estimators. Of course, these estimators do not estimate anything (or, they estimate a "random" object, instead of population parameters), but this approach is discussed in a stationary case; see Davis, R. and Resnick, S., 1985a, Limit theory for moving averages of random variables with regularly varying tail probabilities. Annals of Probability 13, 179–195. Is anything known in a cyclostationary case?

- Typically in time series analysis, the assumption of stable innovations is too restrictive. Does the presented methodology work if stable noise is replaced with regularly varying random variables (such as Pareto, *t*-student

3 Overall Recommendation

Praca spełnia wymagania zawarte w $ 5 ust. 1 i2 Regulaminu:

- Rozprawa doktorska prezentuje ogólną wiedzę teoretyczną kandydata w dyscyplinie oraz umiejętność samodzielnego prowadzenia pracy naukowej.

- Przedmiotem rozprawy doktorskiej jest oryginalne rozwiązanie problemu naukowego.

Ocena rozprawy doktorskiej p. Wojciecha Żuławińskiego jest pozytywna.

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