SUMMARY OF THE DOCTORAL DISSERTATION

Riesz transforms, square functions and Sobolev spaces related to classical and symmetrized Jacobi expansions

by Bartosz Langowski

The dissertation consists of the following author's individual papers.

[L1] *Sobolev spaces associated with Jacobi expansions*,
J. Math. Anal. Appl. 420 (2014), 1533–1551.

[L2] *On potential spaces related to Jacobi expansions*,
J. Math. Anal. Appl. 432 (2015), 374–397.

[L3] *Potential and Sobolev spaces related to symmetrized Jacobi expansions*,
Symmetry, Integrability and Geometry: Methods and Applications; SIGMA 11 (2015), 073, 17 pages.

[L4] *Harmonic analysis operators related to symmetrized Jacobi expansions for all admissible parameters*,
preprint 2015, pp. 1–23, `arXiv:1512.08948`.

In this dissertation Sobolev spaces, potential spaces and fundamental harmonic analysis operators in the context of several types of orthogonal expansions related to the system of classical Jacobi polynomials are studied.

In the paper [L1] one investigates thoroughly the problem of finding a suitable definition of Sobolev spaces in the setting of discrete expansions into Jacobi functions, depending on an appropriate choice of a higher order derivative. The main result [L1, Theorem A] asserts that suitably defined Sobolev spaces are equal, in the sense of isomorphism of Banach spaces, to the corresponding potential spaces. The latter consist of functions belonging to the image of the Lebesgue spaces $L^p(0,\pi)$ under the action of the Jacobi potential operator. This result is an analogue of the classical theorem due to Calderón. Moreover, one studies Sobolev-Jacobi spaces defined by means of a different, seemingly more natural higher order derivative. However, it is shown that this derivative leads to Sobolev spaces that differ from the potential spaces, see [L1, Theorem B].

In the next article [L2] potential spaces in the setting of Jacobi function expansions are investigated. One obtains various results concerning the structure and mutual relations between these spaces ([L2, Theorem 3.1]), as well as a counterpart of the classical Sobolev embedding theorem ([L2, Theorem 3.2]). Furthermore, as the main result of [L2], a characterization of the Jacobi potential spaces in terms of fractional square functions is proved ([L2, Theorems 4.1 and 4.7]). As an important lemma one shows L^p-boundedness of the 'vertical' fractional g-functions associated with Jacobi function and polynomial expansions ([L2, Theorems 6.1 and 6.3]); these results are of independent interest. Finally, sample applications of the Jacobi potential spaces connected with a Cauchy PDE initial value problem based on the Jacobi Laplacian are presented.

The paper [L3] is devoted to the study, in the spirit of [L1, L2], of the setting arising from the symmetrization procedure proposed recently by Nowak and Stempak applied to the context of Jacobi function expansions. Suitable Sobolev and potential spaces are defined and one investigates their properties. The main outcomes of [L3] concern an isomorphism between these spaces [L3, Theorem 3.3] and a characterization of potential spaces by suitably defined fractional square functions [L3, Theorems 4.2 and 4.3]. Moreover, various related results are proved, including structural theorems for the potential spaces and an analogue of Calderón's embedding theorem [L3, Theorems 5.1 and 5.3]. All these results generalize those from [L1, L2] to the symmetrized framework.

The paper [L4] deals with mapping properties of fundamental harmonic analysis operators in the contexts of symmetrized Jacobi polynomial and function expansions. The main result of [L4] says that Riesz transforms, Littlewood-Paley-Stein type square functions, the maximal operator of the Jacobi-Poisson semigroup and multipliers of Laplace and Laplace-Stieltjes transform type in the setting of symmetrized Jacobi polynomial expansions are bounded on weighted L^p spaces for $1 < p < \infty$ and satisfy weighted weak type $(1,1)$ estimate. Similar results were obtained in an earlier author's article, however under a restriction on the associated parameters of type. The paper [L4] releases the results from this restriction. Moreover, the techniques used there lead to shorter and more transparent proofs of the main theorems for the full range of the parameters. Therefore, [L4] should be regarded as an improved and extended version of the aforementioned earlier author's article. Another important achievement of [L4] consists in obtaining similar results in the alternative Jacobi setting arising from considering the system of Jacobi functions, rather than polynomials. This context plays a dominant role in the remaining papers constituting this dissertation.

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