Summary of doctoral dissertation

pt. „Analytic and Borel sets connected with partial orders, ideals, trees and tree ideals"

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The dissertation is devoted to the study of structures related to partial orders, trees, and ideals (in particular, tree ideals).

In Chapter 1, we introduce the basic notions and properties used throughout the rest of the work.

Chapter 2 focuses on the descriptive structure of the discussed objects, more precisely on their descriptive complexity. The main objects of interest will be analytic complete sets, that is, analytic sets which are not Borel. Section 2.1 concerns partial orders and the analytic sets generated by them in a manner analogous to the classical Luzin example of sequences of natural numbers containing a subsequence in which each element divides the next. We will consider similarly defined sets where the divisibility relation is replaced by other relations on countable sets. We will investigate how the properties of these relations affect the descriptive complexity of the resulting sets. We will also, at least partially, characterize the orders for which the resulting sets are non-Borel.

Section 2.2 is devoted to the descriptive complexity of classical ideals of subsets of the set of natural numbers. We proved that the classical Hindman and Ramsey ideals are not Borel. Additionally, we considered new ideals defined in a manner similar to those previously studied and showed their non-Borel nature as well.

Subchapter 2.3 focuses on families of trees. We showed that the set of trees containing Miller/Laver/Silver trees is complete analytic. Furthermore, we considered less classical types of trees, which arose naturally as extensions of classical tree types. We also connected the obtained results with known findings concerning the coding of classical ideals in Polish spaces. In particular, we demonstrated that the set of codes of closed σ-compact sets is complete coanalytic.

In Chapter 3, we consider trees and ideals in the Baire space, which, when viewed as the set of sequences of integers, possesses a natural algebraic structure—coordinatewise addition. These results address the question of whether, for every set from an ideal and every tree of a given type, there exists a subtree of the same type such that the algebraic sum of the ideal set and this subtree remains within the ideal. For meager sets, we obtained a positive answer in the case of perfect trees and negative answers for Miller and Laver trees. Moreover, a key tool in obtaining these results was a new characterization of the ideal of meager sets.

During the study of algebraic sums in the Baire space, natural ideals emerged—namely, a subideal of the meager sets associated with Bartoszyński's characterization in the Cantor space and an ideal imitating sets of measure zero. This inspired the definition and examination of further new ideals in the Baire space, intended to resemble the ideals of meager sets and null sets. In Chapter 4, we will investigate the properties of these ideals—their relationships with each other and with other known ideals, their cardinal invariants, and other set-theoretic properties.