Synopsis of the thesis

Recent rapid advances in single particle tracking and supercomputing techniques resulted in an unprecedented abundance of diffusion data exhibiting complex behaviours, such the presence of power law tails of the mean square displacement and memory functions, commonly referred to as "fractional dynamics" and "anomalous diffusion". It was extensively studied in numerous physical systems, in particular "subdiffusion" was observed in the cytoplasm of living biological cells, various crowded fluids and lipid bilayer membrane systems; and "superdiffusion" was reported in systems related to active biological transport or turbulence. Interesting examples of normal diffusion are also still observed, in which the Brownian-like behaviour can be present together with non-Gaussian distributions or weak ergodicity breaking.

The presented thesis is concerned with studying the role and meaning of the models based on the classical and generalized Langevin equation in modelling of these new, unexplored phenomena. The goal was to present these models from many different perspectives, each with its own advantages and limitations.

The first two chapters briefly sketched the history of the Langevin equation and the basic facts from the theory of Gaussian processes. In the third chapter the Langevin equation was interpreted as a reduced form of Hamilton's equations, which linked macroscopic and microscopic level of physical description. In the fourth chapter, they were translated to the form of discrete-time linear filters. The properties of raw data were stressed, and the presented methods and theorems provided a link between physical models, statistical methods, and experimental observations. The presented techniques were then used to model the optical tweezers data. In the next, fifth chapter, the studied processes were analysed in Fourier space using the spectrum. The dynamics disappeared, and Langevin equations were viewed as multiplicative operators. This allowed to study the somewhat surprising relation between their spectrum and ergodic properties. In the final, sixth chapter, the link between time- and ensemble- averages was explored from a different angle: using the superstatistical approach the Gaussian and ergodic Langevin equations were rendered non-Gaussian, non-ergodic, and thus suitable as models of such types of systems.

The applied methods were primarily based on the theory of Gaussian and second order processes. This was an intentional choice, as this work was mainly concerned in how various models can be distinguished and verified. For this reason the studied properties were mainly those commonly estimated from the data: the covariance function, mean square displacement and power spectral density; similarly the use of the characteristic function was advocated, and it was presented as a tool for analysing non-ergodic and non-Gaussian properties. The usage of Monte Carlo simulations and plots of measurable quantities allowed to illustrate some of the more abstract results, such as the generalisation of the Maruyama theorem.