Asymptotic properties of correlated continuous time random walks

Thesis summary

In the thesis we analyze continuous time random walks $R(t)$ with heavy-tailed waiting times T_i ($i \in \mathbb{N}$)

$$\mathbf{P}(T_i > x) \approx x^{-\alpha}, \quad \text{when} \quad x \to \infty$$

for $\alpha \in (0,1)$. Furthermore we assume that jumps and waiting-times are dependent:

$$J_i = vV_iT_i,$$

where $v > 0$ is a constant which describes the velocity and V_i are random variables which govern the direction of motion. The process $R(t)$ combines two important features: all its moments are finite but the distribution of jumps J_i is heavy-tailed. Moreover $R(t)$ belongs to the class of the so called *Lévy walks* (LW). Lévy walks are useful models of anomalous diffusion and have found many applications. For instance recent fluorescent microscopy experiments have shown that the motion of bacteria can be modeled by LW (*G. Ariel et al., Swarming bacteria migrate by Lévy Walk, Nature Communications 2015*). There is also an important hypothesis - *Lévy flight foraging hypothesis* which says that Lévy walks and flights are optimal search strategies when prey is sparse.

We analyze properties of the diffusion limit process

$$\frac{1}{n}R(nt) \xrightarrow[n\to\infty]{J_1} X_t,$$

where $\xrightarrow[n\to\infty]{J_1}$ denotes weak convergence in J_1 Skorokhod space. The limit process X_t has a complicated structure which include α-stable processes and their inverse. Apart from the process $R(t)$ we also analyze two different types of Lévy walks and their limit processes Y_t and Z_t. Below are listed main results of the thesis.

- We described path properties (total variation, asymptotic behavior) and martingale properties of the limit processes X_t, Y_t and Z_t.
- We calculated probability density functions of the limit processes. This was done using two different methods. We also obtained solutions of certain differential equations with a fractional material derivative.
- Two point distributions (X_{t_1}, X_{t_2}) , for $t_2 > t_1 > 0$ were determined using semi-Markov approach. A similar method can be used to find all finite dimensional distributions.
- The distribution of increments of the limit processes was explicitly derived. We also investigated the effect of aging on MSD and TAMSD.
- We analyzed multidimensional isotropic Lévy walks and their limits. We developed a new method to find explicit expressions for their PDFs. It turns out that when the number of dimensions is odd the result is given by elementary functions.
- Explicit formulas for potential measure of certain nonsymmetric α-stable processes connected with the limit processes of Lévy walks were derived.
- We have proven that Lamperti transformation and increments of Lévy flights coexisting with subdiffusion are ergodic. We also have shown a similar result for the subordinated fractional Brownian motion.