

FACULTY OF PURE AND APPLIED MATHEMATICS

**SUBJECT CARD**

**Name of subject in Polish:** Zaawansowane zagadnienia z teorii gier dynamicznych

**Name of subject in English:** Advanced Topics in Dynamic Games

**Main field of study (if applicable):** Applied Mathematics

**Specialization (if applicable):** Modelling, Simulation, Optimization

**Profile:** academic / practical\*

**Level and form of studies:** 2nd level/ full-time/

**Kind of subject:** optional

**Subject code**

**Group of courses** YES

	Lecture	Classes	Laboratory	Project	Seminar
Number of hours of organized classes in University (ZZU)	30	30			
Number of hours of total student workload (CNPS)	90	60			
Form of crediting	crediting with grade				
For group of courses mark (X) final course	X				
Number of ECTS points	3	2			
including number of ECTS points for practical classes (P)	2	2			
including number of ECTS points corresponding to classes that require direct participation of lecturers and other academics (BU)	3				

\*delete as not necessary

**PREREQUISITES RELATING TO KNOWLEDGE, SKILLS AND OTHER COMPETENCES**

1. Student has a basic knowledge of the calculus, algebra and the probability theory.
2. Student has a basic knowledge of game theory.

**SUBJECT OBJECTIVES**

- C1. Basic knowledge of Markov decision processes.
- C2. Basic knowledge of algorithms allowing to find value functions and optimal policies.
- C3. Basic knowledge of simple markovian decision models.
- C4. Basic knowledge of stochastic game models.
- C5. Basic knowledge of theory and applications of mean field games.
- C6. Ability to apply the acquired knowledge to create and analyze dynamic optimization models in various fields of science and technology.

**SUBJECT EDUCATIONAL EFFECTS**

*The scope of the student's knowledge:*

PEU\_W01 Students knows basic concepts of dynamic programming.

PEU\_W02. Student knows basics of theory of stochastic games.  
 PEU\_W03. Student knows basics of theory of mean field games.

The scope of the student's skills:

PEU\_U01 Student is able to find an optimal policy and value function in a simple markovian decision process.  
 PEU\_U02. Student is able to check whether a vector of strategies forms a Nash equilibrium in a given simple stochastic game.  
 PEU\_U03. Student is able to construct an appropriate dynamic model of a given optimization problem.

The scope of the student's social skills:

PEU\_K01. Student is able to utilise literature pointed out by the lecturer.  
 PEU\_K02. Student is able to use computer programs in order to solve some issues.  
 PEU\_K03. Student understands the necessity of further self-learning.

<b>PROGRAMME CONTENT</b>		
<b>Lecture</b>		<b>Number of hours</b>
Lec1	Introduction to markovian decision processes, the concept of a policy, different optimality criteria, examples of simple models.	2
Lec2	Dynamic programming method. Solving models with finite time horizon. Backward induction.	2
Lec3	Models with infinite time horizon. The Banach fixed point theorem and its application to a solution of the Bellman equation.	2
Lec4	Algorithms applied to infinite time horizon models: value iteration, policy improvement, LP.	4
Lec5	Markov decision processes with risk sensitive payoff criteria. Other payoff criteria.	2
Lec6	Specific models.	2
Lec7	Two-person zero-sum discounted stochastic games. The theorem of Shapley.	4
Lec8	Nonzero-sum discounted stochastic games.	2
Lec9	Stochastic games with other payoff criteria.	2
Lec10	Applications of stochastic games in economics and engineering.	2
Lec11	Mean field games. The existence of solutions. Relation with games with a finite number of players. Examples of applications in economics and engineering.	4
Lec12	Summary and exam.	2
<b>Total hours</b>		<b>30</b>

<b>Classes</b>		<b>Number of hours</b>
Cl 1	Markov chains.	2
Cl 2	Solving different markovian decision models.	14
Cl 3	Solving different stochastic game models.	14
<b>Total hours</b>		<b>30</b>

### TEACHING TOOLS USED

- N1. Lecture – traditional method  
 N2. Exercise and accounting problems - the traditional method.  
 N3. Consultation.  
 N4. Student's own work - preparing to exercise and test.

### EVALUATION OF SUBJECT LEARNING OUTCOMES ACHIEVEMENT

Evaluation (F – forming (during semester), P – concluding (at semester end))	Educational effect number	Way of evaluating educational effect achievement
F1	PEK_W01,PEK_W02, PEK_W03,  PEK_U01,PEK_U02,  PEK_U03,PEK_K01,  PEK_K02	oral presentations, quizzes
F2	PEK_W01,PEK_W02,  PEK_U01,PEK_U02,  PEK_U03,PEK_K01,  PEK_K02	exam
P=0,5*F1+0,5*F2		

### PRIMARY AND SECONDARY LITERATURE

#### **PRIMARY LITERATURE:**

- [1] M. Puterman, Markov decision processes, Wiley 1994.  
 [2] N. Stockey, R. Lucas, E. Prescott, Recursive methods in economic dynamics, Harvard University Press, 1989.  
 [3] A. Haurie, J.B. Krawczyk, G. Zaccour. Games and Dynamic Games. World Scientific, 2012.

#### **SECONDARY LITERATURE:**

- [4] H. Tijms, A first course in stochastic models, Wiley 2003.  
 [5] B. Jovanovic and R. W. Rosenthal. Anonymous sequential games. Journal of Mathematical Economics, 17:77–87, 1988.  
 [6] O. Gueant, J-M. Lasry, P-L. Lions, Mean field games and applications. W R. Carmona et al., editor, Paris Princeton Lectures in Mathematical Finance IV, Lecture Notes in Mathematics v.2003. Springer Verlag, 2010.

#### **SUBJECT SUPERVISOR (NAME AND SURNAME, E-MAIL ADDRESS)**

Dr hab. inż. Anna Jaskiewicz (Anna.Jaskiewicz@pwr.edu.pl)  
 Dr Piotr Więcek (Piotr.Wiecek@pwr.edu.pl)