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Fractional models and their applications in finance

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Abstract of doctoral dissertation

The doctoral dissertation focuses on the subdiffusive and tempered subdiffusive Black-Scholes models. In contrast to their classical equivalent, they can be used in markets where periods of stagnation are observed. The main part of the work consists of the analysis of the finite difference method as a numerical approach to the option pricing in the considered models. The governing fractional differential equations and the related weighted numerical schemes are derived. The proposed methods have $2-\alpha$ order of accuracy with respect to time, where $\alpha \in (0,1)$ is the subdiffusion parameter, and 2 with respect to space. Furthermore, the consistency, stability, and convergence analysis are provided. Moreover, the optimal choice of the weighting parameter θ is discussed. In the subdiffusive case, the optimal numerical scheme is a generalization of the Crank-Nicolson method. Furthermore, a general case of subordination of the classical option pricing models is considered. The relation between Bachelier and Black-Scholes models driven by the inverse subordinators is investigated. The subordinated Cox-Ross-Rubinstein model is introduced and it is shown that the price of the underlying in that model converges in distribution and in Skorokhod space to the price of underlying in the subordinated Black-Scholes model. Motivated by this fact and the analogous result for the option prices, the selected option contracts are priced using binomial trees. The numerical results are also provided.

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